Reg. No. : [ ]

Question Paper Code : 25141

B.E./B.Tech. DEGREE EXAMINATION, NOVEMBER/DECEMBER 2018.

Third Semester

Civil Engineering

MA 8353 – TRANSFORMS AND PARTIAL DIFFERENTIAL EQUATIONS

(Common to: Electrical and Electronics Engineering / Industrial Engineering and Management / Chemical and Electrochemical Engineering / Aeronautical Engineering / Agriculture Engineering / Automobile Engineering / Electronics and Instrumentation Engineering / Industrial Engineering / Instrumentation and Control Engineering / Manufacturing Engineering / Marine Engineering / Material Science and Engineering / Mechanical Engineering / Mechanical Engineering (Sandwich) / Mechanical and Automation Engineering / Mechatronics Engineering / Production Engineering / Robotics and Automation Engineering / Bio Technology/ Food Technology and Pharmaceutical Technology)

(Regulations 2017)

Time : Three hours — Maximum : 100 marks

Answer ALL questions.

PART A — ($10 \times 2 = 20$ marks)

1. What are singular integrals? How does it differ from particular integral?

2. Solve $2\dfrac{\partial^2 z}{\partial x^2} + 5\dfrac{\partial^2 z}{\partial x \partial y} + 2\dfrac{\partial^2 z}{\partial y^2} = 0$.

3. What is the behavior of Fourier series of a function $f(x)$ at the point of discontinuity?

4. Sketch the even and odd extensions of the periodic function $f(x) = x^2$ for $0 < x < 2$.

5. Classify the partial differential equation $2x\dfrac{\partial^2 u}{\partial x^2} + 4x\dfrac{\partial^2 u}{\partial x \partial y} + 8x\dfrac{\partial^2 u}{\partial y^2} = 0$

6. Mention the various possible general solutions for one dimensional heat equation.

7. Does Fourier sine transform of $f(x) = k, 0 \le x \le \infty$, , exist? Justify your answer.

8. State convolution theorem for Fourier transforms.

9. What are the applications of Z-Transform?

10. Find the Z transform of $f(n) = (n+1)^2$.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Form the partial differential equation by eliminating the arbitrary function from $f(x^2 + y^2, z - xy) = 0$. (8)

(ii) Find the solution of the partial differential equation

$$\frac{\partial^2 z}{\partial x^2} - 4x\frac{\partial^2 z}{\partial x \partial y} + 4\frac{\partial^2 z}{\partial y^2} = e^{2x+y}.$$ (8)

Or

(b) (i) Solve the Lagrange's linear equation
$$(x^2 - yz)p + (y^2 - zx)q = z^2 - xy.$$ (8)

(ii) Solve the partial differential equation

$$(D^2 + 2DD' + D'^2 - 2D - 2D')z = \sin(x + 2y).$$ (8)

12. (a) (i) Obtain the Fourier series of the periodic function $f(x) = e^{ax}$ in the interval $0 \le x \le 2\pi$. (8)

(ii) Develope the Fourier series for the function $f(x) = \begin{cases} 1 + \frac{2x}{\pi}, & -\pi \le x \le 0 \\ 1 - \frac{2x}{\pi}, & 0 \le x \le \pi \end{cases}$

Hence deduce that $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + ... = \frac{\pi^2}{8}$. (8)

Or

(b) (i) Find the complex form of the Fourier series for $f(x) = e^{-x}$, in $-1 \le x \le 1$. (8)

(ii) Develope the half range Fourier series for the function $f(x) = x^3$ in $(0, L)$. (8)

13. (a) (i) Using the method of separation of variables solve $\frac{\partial u}{\partial x} = 2\frac{\partial u}{\partial t} + u$, where $u(x,0) = 6e^{-3x}$. (8)

(ii) Find the temperature $u(x,t)$ in a laterally insulated heat conducting bar of length L with its ends kept at 0° and with the initial temperature in the bar is $u(x,0) = 100\sin\left(\frac{\pi x}{80}\right)$ and $L = 80\ cm$. (8)

Or

(b) (i) Derive the general solutions for one dimensional wave equation $\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$ using separation of variables method. (8)

(ii) Find the displacement of a string stretched between two fixed points at a distance L apart. The string is initially at rest in equilibrium position and points of the string are given initial displacement $u(x,0) = k(Lx - x^2)$. Assume initial velocity zero. (8)

14. (a) (i) Find the Fourier transform of $f(x) = \begin{cases} 1 - x^2, & |x| \le 1 \\ 0, & |x| \ge 1 \end{cases}$. Hence deduce $\int_0^\infty \frac{x \cos x - \sin x}{x^3} \cos\left(\frac{x}{2}\right) dx$. (10)

(ii) Construct the Fourier sine transform of $f(x) = \frac{e^{-ax}}{x}$. (6)

Or

(b) (i) Find the Fourier cosine transforms of $f(x) = e^{-ax}$ and $g(x) = e^{-bx}$. Using these transforms and Parseval's identity show that $\int_0^\infty \frac{dt}{(a^2 + t^2)(b^2 + t^2)} = \frac{\pi}{2ab(a+b)}$. (10)

(ii) Find the Fourier transform of $f(x) = \cos x,\ 0 \le x \le 1$. (6)

15. (a) (i) Form the difference equation corresponding to the family of curves $y = ax + bx^2$. (8)

(ii) Find the Z transform of $u(n) = 3n - 4\sin\left(\frac{n\pi}{4}\right) + 5a$, and $u(n) = \cos\left(\frac{n\pi}{2} + \frac{\pi}{4}\right)$. (8)

Or

(b) (i) Use convolution theorem to evaluate the inverse Z transform of $U(z) = \frac{z^2}{(z-a)(z-b)}$. (6)

(ii) Solve the difference equation $y_{n+2} + 6y_{n+1} + 9y_n = 2^n$ with initial conditions $y_0 = y_1 = 0$, using Z transform. (10)

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